Supercurrent transferring through *c*-axis cuprate Josephson junctions with thick normal-metal bridge

Supercurrent transferring through c-axis cuprate Josephson junctions with thick normal-metal bridge

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Abstract
With a simple but exactly solvable model, we investigate the supercurrent transferring through
the c-axis cuprate superconductor–normal metal–superconductor junctions with the clean
normal metal much thicker than its coherence length. It is shown that the supercurrent as a
function of thickness of the normal metal decreases much slower than the exponential decaying
expected by the proximity effect. The present result may account for the giant proximity effect
observed in the c-axis cuprate SNS junctions.

(Some figures in this article are in colour only in the electronic version)

In a superconductor–normal metal junction, it is considered that the Cooper pair can penetrate into the normal metal within a distance of the coherence length ξ_n due to the proximity effect [1]. Therefore, according to the proximity theory, the supercurrent cannot transfer through a superconductor–normal metal–superconductor (SNS) junction when the normal metal is much thicker than ξ_n. However, the supercurrent in high-temperature-superconductor (HTSC) junctions with very thick barrier (consisting of weakly doped nonsuperconducting cuprates) has been observed by a number of experiments [2–9]. There have been some theoretical explanations based on the assumption of the existence of superconducting puddles in the pseudogap states of the cuprates [10, 11]. But the physics of pseudogap states of the cuprates is not clearly understood so far. The explanation based on the tunneling of the preformed pairs in cuprates has also been proposed recently [12]. Nevertheless, the problem whether the supercurrent can transfer through a long bridge of normal metal between two superconductors is still an outstanding puzzle.

The supercurrent stems from the motion of paired carriers in the superconductor. It is known that the supercurrent can be conducted by Andreev reflections in the SNS junctions [13–24]. The supercurrent in one superconductor, for example in the left one, can transfer through the SN interface by generating the propagations of electrons and holes in the normal metal due to the Andreev refection. At another NS interface, the electrons and holes are converted

into paired electrons in the right superconductor [25]. As a result, the paired particles are conducted from the left to right superconductors even though Cooper pairs cannot survive in the normal metal. In the case of a clean normal metal without large damping in particle propagations, the supercurrent may transfer through the long SNS junction. A study of the supercurrent in the c-axis cuprate SNS junctions of a thick normal-metal bridge with this approach is still necessary.

In this work, on the basis of the Andreev-reflection approach, we study the supercurrent in the c-axis cuprate SNS junctions using a simple but exactly solvable model. We will show that the supercurrent can transfer through the junctions with the normal metal much thicker than its coherence length. We intend to provide a possible explanation for the relevant experiments.

We consider a c-axis cuprate SNS junction with the normal metal occupying the layers from $1-l$ to $l-1$. A sketch of the junction is shown in figure 1. Within the ab plane, the quasiparticles are described by the $t - t'$ tight-binding model with $t'/t = -0.3$. The phase difference ϕ between the pair potentials of the two superconductors drives the supercurrent. With an unitary transformation $\hat{U} = \exp(i\sigma_3\phi/4)$, one can show that the physical quantities of the system depend only on the total phase difference of the pair potentials. For convenience, we here set the phases of the pair potentials as $\pm\phi/2$, respectively, for the left and right superconductors. The electron motion along the c axis is described by the interlayer
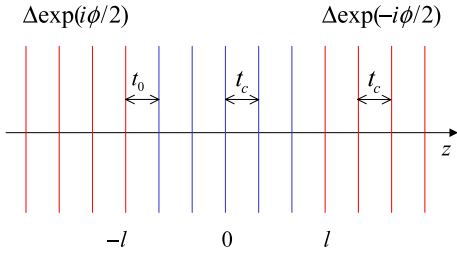
Figure 1. Sketch of a c-axis cuprate SNS junction. The interlayer hopping is described by t_c. t_0 is the hopping through the NS interface.

hopping t_c with $t_c/t \ll 1$. The magnitude of t_c may be of the same order as Δ. The parameter of electron hopping through the SN interface is t_0. Throughout this paper, we use the units of $e = \hbar = 1$ with $-e$ as the charge of an electron.

The states of the quasiparticles are described by the Bogoliubov–de Gennes (BdG) equation [26]. Since the momentum parallel to the interfaces is conserved during the motion of the quasiparticles through the junction, the transverse (orthogonal to the c axis) part of the wavefunction can be taken as plane waves. The problem is then reduced to solving the one-dimensional BdG equation along the z direction. For an eigenstate of transverse momentum $k_\perp$, the chemical potential in the BdG equation is then substituted by $\tilde{\mu}(k_\perp) = \mu - \epsilon(k_\perp)$, where $\epsilon(k_\perp) = -2t(\cos k_1 + \cos k_2) - 4t' \cos k_1 \cos k_2$ is the in-plane single-particle energy, with k_1 and k_2 the two components of $k_\perp$. The order parameters of the superconductors are given by $\Delta(k_\perp) \exp(\pm i\phi/2)$ (+ and − for left and right superconductors, respectively) with $\Delta(k_\perp) = \Delta(\cos k_1 - \cos k_2)$. The BdG equation is

$$\sum_j H_{ij}\psi(j) = E\psi(i), \tag{1}$$

with

$$H_{ij} = \begin{cases} [v_i - \tilde{\mu}(k_\perp)]\sigma_3 + \Delta_i\sigma^+ + \Delta_i^*\sigma^-, & \text{for } i = j \\ -t_c\sigma_3, & \text{for nearest-layer hoppings} \\ -t_0\sigma_3, & \text{for interface hoppings} \end{cases}$$

where $v_i = V_0$ for $1 - l < i < l - 1$ or 0 otherwise, $\Delta_i = \Delta(k_\perp) \exp(i\phi/2)$ for $i \leqslant -l$, $\Delta_i = \Delta(k_\perp) \exp(-i\phi/2)$ for $i \geqslant l$, and σ's are the Pauli matrices. The potential shift V_0 controls the density difference between the normal metal and the superconductors. All the states in a complete basis can be divided into three types: incoming waves of free states from the left and right superconductors, and the bound states mainly confined in the normal metal with damping tails in the two superconductors.

The free state with an incoming wavenumber k^+ from the left superconductor is obtained as [19]

$$\psi_{l1}(j) = \begin{pmatrix} ue_\phi \\ ve_\phi^* \end{pmatrix} (e^{ik^+z'} + be^{-ik^+z'}) + a \begin{pmatrix} ve_\phi \\ ue_\phi^* \end{pmatrix} e^{ik^-z'},$$

$$z' = j + l \leqslant 0$$

$$\psi_{l2}(j) = \begin{pmatrix} A_1e^{iq_1 j} + A_2e^{-iq_1 j} \\ B_1e^{-iq_2 j} + B_2e^{iq_2 j} \end{pmatrix}, \qquad 1 - l \leqslant j \leqslant l - 1$$
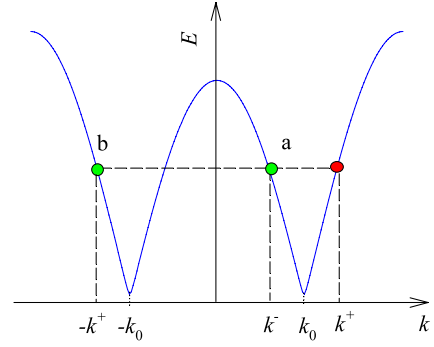
Figure 2. Sketch for the definition of wavenumbers k^+ and k^- on energy curve $E(k)$. The reflected waves of an incoming wave of k^+ include two components of a, the Andreev, and b, the normal, reflections.

$$\psi_{l3}(j) = c \begin{pmatrix} ve_\phi^* \\ ue_\phi \end{pmatrix} e^{-ik^-z'} + d \begin{pmatrix} ue_\phi^* \\ ve_\phi \end{pmatrix} e^{ik^+z'},$$

$$z' = j - l \geqslant 0$$

with the boundary conditions

$$r\psi_{l1}(-l) - \psi_{l2}(-l) = 0$$
$$\psi_{l1}(1 - l) - r\psi_{l2}(1 - l) = 0$$
$$r\psi_{l2}(l - 1) - \psi_{l3}(l - 1) = 0 \tag{2}$$
$$\psi_{l2}(l) - r\psi_{l3}(l) = 0$$

where $e_\phi = \exp(i\phi/4)$, the wavenumbers q_1, q_2, k^+ and k^- satisfy the equations $\xi(q_1) + V_0 = -\xi(q_2) - V_0 = \sqrt{\xi^2(k^+) + \Delta^2(k_\perp)} \equiv E_k$ and $\xi(k^-) = -\xi(k^+)$ with $\xi(k) = -2t_c \cos k - \tilde{\mu}(k_\perp)$, $u = \sqrt{1/2 + \xi(k^+)/2E_k}$, $v = \sqrt{1/2 - \xi(k^+)/2E_k}$ and $r = t_0/t_c$. The incoming wavenumber k^+ is defined in the ranges $-k_0 < k^+ < 0$ and $k_0 < k^+ < \pi$, where the group velocity $\partial E_k/\partial k$ is positive, with $k_0 = \arccos[-\tilde{\mu}(k_\perp)/2t_c]$ as the 'Fermi wavenumber' along the z direction. A sketch for the definition of k^+ and k^- is shown in figure 2. The eight coefficients $a, b, \ldots$ are determined by the boundary conditions (2). Denoting

$$X^t = (a, b, A_1, B_1, c, d, A_2, B_2),$$

with superscript t implying the transpose of vector X, we have from equations (2)

$$MX = Z, \tag{3}$$

where M is an 8×8 matrix and Z is a column vector of 8 components. Expressing M in terms of 16-block 2×2 matrices, we get

$$M = \begin{pmatrix} D_1(\phi) & O(-l) & 0 & O(l) \\ D_2(\phi) & rO(1-l) & 0 & rO(l-1) \\ 0 & rO(l-1) & D_2(-\phi) & rO(1-l) \\ 0 & O(l) & D_1(-\phi) & O(-l) \end{pmatrix}, \tag{4}$$

$$D_1(\phi) = \begin{pmatrix} rve_\phi & rue_\phi \\ rue_\phi^* & rve_\phi^* \end{pmatrix},$$

$$D_2(\phi) = \begin{pmatrix} ve_\phi e_- & ue_\phi e_+^{-1} \\ ue_\phi^* e_- & ve_\phi^* e_+^{-1} \end{pmatrix},$$

$$O(l) = \begin{pmatrix} -e_1^l & 0 \\ 0 & -e_2^{-l} \end{pmatrix},$$

with $e_\pm = \exp(ik^\pm)$. The vector Z is given by

$$Z^t = (-rue_\phi, -rve_\phi^*, -ue_\phi e_+, -ve_\phi^* e_+, 0, 0, 0, 0).$$

Similarly, we can write down the expression for the incoming wavefunctions ψ_r from the right superconductor. But with the configuration of the SNS junction under consideration, ψ_r can be obtained from the relation

$$\psi_r(j; \phi) = \lambda \psi_l(-j; -\phi), \tag{5}$$

with $\lambda = \pm 1$.

For the wavefunction ψ_n of a bound state with energy $0 < E_n < |\Delta(k_\perp)|$, the expression is given by [23]

$$\psi_{n1}(j) = a_n \begin{pmatrix} u_n^* e_\phi \\ u_n e_\phi^* \end{pmatrix} e^{ik^* z'} + b_n \begin{pmatrix} u_n e_\phi \\ u_n^* e_\phi^* \end{pmatrix} e^{-ikz'},$$
$$z' = j + l < 0$$

$$\psi_{n2}(j) = \begin{pmatrix} A_1^n e^{iq_1^n j} + A_2^n e^{-iq_1^n j} \\ B_1^n e^{-iq_2^n j} + B_2^n e^{iq_2^n j} \end{pmatrix}, \qquad -l < j < l$$

$$\psi_{n3}(j) = c_n \begin{pmatrix} u_n^* e_\phi^* \\ u_n e_\phi \end{pmatrix} e^{-ik^* z'} + d_n \begin{pmatrix} u_n e_\phi^* \\ u_n^* e_\phi \end{pmatrix} e^{ikz'},$$
$$z' = j - l > 0$$

where k is a complex wavenumber determined by $\xi(k) = i\gamma$ with $\gamma = \sqrt{\Delta^2(k_\perp) - E_n^2}$ (Im $k > 0$), $u_n = \exp(i\theta/2)/\sqrt{2}$ with $\theta = \arctan(\gamma/E_n)$, q_1^n and q_2^n are determined by $\xi(q_1^n) + V_0 = -\xi(q_2^n) - V_0 = E_n$. The vector of the coefficients

$$X_n^t = (a_n, b_n, A_1^n, B_1^n, c_n, d_n, A_2^n, B_2^n),$$

now satisfies the following equation:

$$M_n X_n = 0, \tag{6}$$

where M_n is a counterpart of M with u, v, k^+, k^-, q_1 and q_2 replaced with $u_n, u_n^*, k, k^*, q_1^n$ and q_2^n, respectively. The energy E_n is then determined by

$$\det(M_n) = 0. \tag{7}$$

The solution to the jth component of X_n is given by the algebraic complement minor of 1jth element of M_n (multiplied by a factor that is determined by the normalization condition $\langle \psi_n | \psi_n \rangle = 1$). We note at this moment that $(a_n, b_n, A_1^n, B_1^n) = \pm(c_n, d_n, A_2^n, B_2^n)$ at $\phi = 0$ because of $\psi_n(x; \phi) = \pm\psi_n(-x; -\phi)$. Therefore, for a finite phase difference, the coefficients a_n, b_n, A_1^n, B_1^n should have respectively the same orders of magnitudes of c_n, d_n, A_2^n, B_2^n. There are various approximations based on the Andreev and WKB approximations in the existing theories [14–17]. The approximation in [16] corresponds to $b_n = c_n = A_2^n = B_1^n = 0$, taking into account only the Andreev reflections but neglecting the normal reflections at the right NS interface. It is not correct. Actually, in a bound state, the electrons

and holes are bounced back and forth again and again in the normal metal. The normal and Andreev reflections at the two interfaces are equally important.

To derive the expression of the current, we start from the operator of current density in the continuum model of the normal metal:

$$J(x) = -\text{Im}[\psi^\dagger(x)\nabla\psi(x)]. \tag{8}$$

For the lattice case, $\nabla\psi(x)$ in equation (8) is replaced with $[\psi(j+1) - \psi(j-1)]/2$. Taking the statistical average by summing up all the contributions from the states of positive and negative energies, we obtain

$$J = \int_{BZ} \frac{d\vec{k}_\perp}{(2\pi)^2} \left[\int \frac{dk^+}{2\pi} \tanh\left(\frac{E_k}{2T}\right) J_f(\vec{k}) \right.$$
$$\left. + \sum_n \tanh\left(\frac{E_n}{2T}\right) \text{Re}(A_+^{n*} A_-^n \sin q_1^n - B_+^{n*} B_-^n \sin q_2^n) \right] \tag{9}$$

with

$$J_f(k) = \text{Re}\{[A_+^*(\phi)A_-(\phi) - A_+^*(-\phi)A_-(-\phi)]\sin q_1$$
$$- [B_+^*(\phi)B_-(\phi) - B_+^*(-\phi)B_-(-\phi)]\sin q_2\}.$$

where the integral $\int_{BZ} d\vec{k}_\perp$ runs over the first Brillouin zone, $A_\pm = A_1 \pm A_2$, and T is the temperature. The first term J_f on the right-hand side of equation (9) comes from the contributions of the free states. The second term is due to the bound states. Here, the phase dependence of the coefficients of the free waves is explicitly indicated by ϕ as their argument. Of course, those coefficients of the bound states A^n's and B^n's, the energy E_n and the wavenumbers q^n's depend on the phase ϕ as well. At $\phi = 0$, corresponding to the equilibrium state, there is no current flowing through the junction. The current is driven by a finite phase difference. Instead, to investigate the phase dependence, we here confine ourselves to the problem of length $L = 2l$ dependence of the supercurrent with fixed phase difference $\phi = \pi/2$.

For numerical calculation, we need to first determine the parameters t_c, t_0, μ, V_0 and Δ. The electron hopping through the SN interface was chosen as $t_0 = 0.8t_c$. The chemical potential μ and the potential shift V_0 in the normal metal were set respectively to $\mu/t = -0.97$ and $V_0/t = -0.042$, corresponding to the hole densities $\delta_s \approx 0.13$ in the superconductor and $\delta_n \approx 0.11$ in the normal metal (at finite temperature). Shown in figure 3 are the calculated results for the supercurrent as a function of the distance $L = 2l$ (in units of the c-direction lattice constant) between two superconductors. The circles and squares correspond to parameters of $(t_c, \Delta, T)/t = (5, 4.75, 0.25) \times 10^{-2}$ and $(6, 3.8, 0.2) \times 10^{-2}$, respectively. For comparison, we also depict the curves of $J(L) \propto \exp(-L/\xi)$ with $\xi = 4$ and 6. Each curve does not match the corresponding numerical results so well. In particular, at large L, the behavior of $J(L)$ obtained by the present calculation shows that the current decays much slower than the exponential type. On the other hand, by the proximity theory, $J(L)$ should decay exponentially with a much shorter coherence length $\xi_n \sim 1$–2 Å for cuprates [9]. In units of the c-axis constant c, this typical coherence length ξ_n is
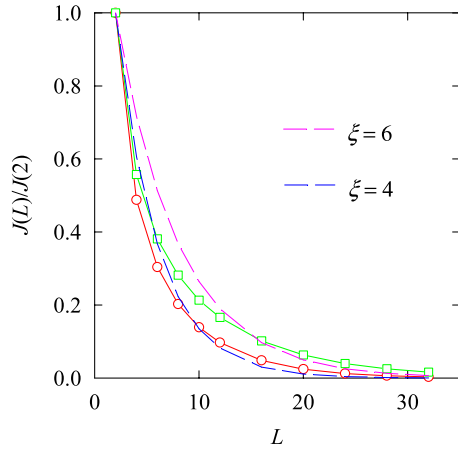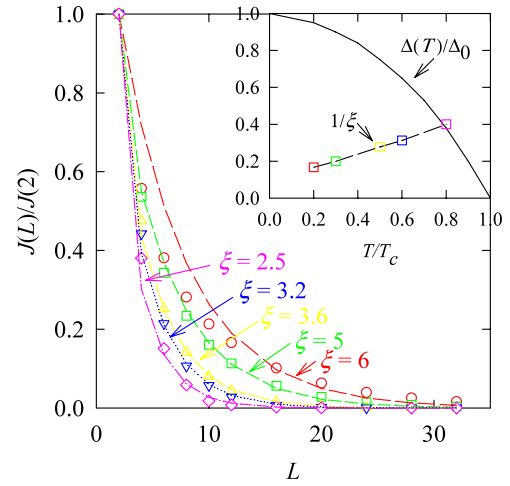
Figure 3. c-axis supercurrent J through a d-wave SNS junction as a function of the distance L between two superconductors. L is given in units of the c-axis lattice constant. The circles and squares are the results for two sets of parameters $(t_c, \Delta, T)/t = (5, 4.75, 0.25)$ $\times 10^{-2}$ and $(6, 3.8, 0.2) \times 10^{-2}$, respectively. The dashed lines express the formula $\exp[-(L - 2)/\xi]$ with $\xi = 4$ and $\xi = 6$, respectively.

Figure 4. c-axis supercurrent J through a d-wave SNS junction as a function of the distance L between two superconductors at $T/T_c = 0.2, 0.3, 0.5, 0.6$ and 0.8. The symbols are the calculated results. The dashed lines express the formula $\exp[-(L - 2)/\xi]$. The inset shows the order parameter $\Delta(T)$ and the inverse of ξ as functions of T.

about 0.1–0.2 if $c = 10$ Å. Our model is the simple rectangular parallelepiped crystal corresponding to the orthogonal system like HgBaCuO, for which c is 9.5 Å. Most of the cuprates are staggered stacking and there are multiple layers in each unit cell. The averaged c is shorter and ξ_n is estimated as 0.2–0.5. Our result implies that the supercurrent can flow through the junction with $L > \xi \gg \xi_n$.

For investigating the temperature dependence, we put the order parameter $\Delta(T)$ as an overall function of T as shown in the inset of figure 4 with $\Delta_0/T_c = 4$ and 0.04. The parameter t_c is fixed as $t_c/t = 0.06$. Figure 4 shows the results for the supercurrent as a function of the distance L between two superconductors at various temperatures. They are compared with the formula $\exp[-(L - 2)/\xi]$. At $T/T_c \geqslant 0.3$, the numerical results are well fitted by the exponential forms. The inverse of ξ as a function of T is also shown in the inset of figure 4. The circles and the dashed line in figure 4 are the results at $T/T_c = 0.2$ and correspond to the squares and the dashed line of the same $\xi = 6$ in figure 3, respectively. These results show that the supercurrent can flow through a junction much thicker than ξ_n. In particular, at low temperature, the current decays much slower than the exponential type at large L.

To compare the present calculation with the proximity theory for the layered system, we here estimate the theoretical coherence length ξ_c along the c axis [7]. According to the uncertainty principle, ξ_c is proportional to the inverse of the uncertainty of the momentum δp_c of electrons. The latter can be estimated as $v_c \delta p_c \approx \delta E$, where v_c is the averaged magnitude of the electron velocity along the c direction. Note that there is no Fermi surface across the c axis in the layered system with weak interlayer hopping. From the energy dispersion in the c direction, $\epsilon(q) = -2t_c \cos(q)$ with q the c-axis momentum in units of c (c-axis lattice constant) $= 1$, we obtain the electron velocity $2t_c \sin(q)$. The overall magnitude

of v_c can be estimated as t_c. On the other hand, the uncertainty of the energy ΔE is the order of the bandwidth $4t_c$. We then have $\xi_c \approx 1/4$. This ξ_c is approximately the same as the observed data for cuprates. Therefore, according to the proximity theory, the supercurrent cannot transfer along the c axis even for very short SNS junctions. However, since the supercurrent can be conducted by the Andreev reflections, it is not limited by the coherence length. Our calculation may account for the giant proximity effect observed in the cuprate SNS junctions.

In summary, we have investigated the supercurrent transferring through the c-axis cuprate SNS junctions. Due to the Andreev reflections, the supercurrent is conducted by the in-gap bound states and the free states above but close to the gap. It is shown that the supercurrent as a function of thickness of the normal metal decreases much slower than the exponential decaying expected by the proximity effect. This result implies that the supercurrent can transfer through the clean c-axis cuprate SNS junctions with the normal metal much thicker than its coherence length. The present result may account for the giant proximity effect observed in the cuprate SNS junctions.

Acknowledgments

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